Hausdorff School on

"Stability Conditions in Representation Theory"

9 to 13 September 2019

organized by
Gustavo Jasso, Jan Schröer

Abstracts

Lecture Series

Dylan Allegretti (Mathematical Sciences Research Institute (MSRI))

Bridgeland stability and related topics

Abstract: The notion of a stability condition on a triangulated category was introduced by Tom Bridgeland around fifteen years ago to formalize ideas about D-branes in string theory. Today, Bridgeland stability conditions have a range of important applications in different parts of mathematics and mathematical physics. In this lecture series, I will give an introduction to Bridgeland stability conditions and describe some recent exciting developments in the subject.
Lecture 1. I will describe stability conditions on an important class of triangulated categories constructed from quivers with potential. A particularly interesting set of examples comes from considering triangulations of surfaces.
Lecture 2. I will review the work of Bridgeland and Smith, which identifies the stability conditions of the previous lecture with meromorphic quadratic differentials on Riemann surfaces.
Lecture 3. I will discuss a recent idea of Bridgeland in which a Riemann-Hilbert problem is naturally associated to a stability condition. This leads to an interesting analogy between the theory of stability conditions and the theory of ordinary differential equations.

Sota Asai (Research Institute for Mathematical Sciences (RIMS), Kyoto University)

The wall-chamber structures for path algebras

Abstract: In this talk, I will mainly deal with the wall-chamber structures comingfrom stability conditions for path algebras. To each module M over a path algebra A, we associate the subset Θ_M of the real Grothendieck group $K_0(\operatorname{proj} R$ in terms of stability conditions, and by regarding such Θ_M as walls, we get a wall-chamber structure of the real Grothendieck group. For a dimension vector d, a result by Schofield implies that the union $\Theta_d := \bigcup_X \Theta_X$ (X runs over the set of all modules whose dimension vector is d) of the walls is actually realized by some module M. I have shown that the walls Θ_d satisfy a simple recurrence relation in my preprint arXiv:1905.02180. I would like to tell you about the details of this result with basic properties on wall-chamber structures in my talk. Moreover, I hope that I can give you some further progress after the preprint.

Jenny August (University of Edinburgh)

The Stability Manifold of Contraction Algebras

Abstract: For a finite dimensional algebra, Bridgeland stability conditions can be viewed as a continuous generalisation of tilting theory, providing a geometric way to study the derived category. Describing this stability manifold is often very challenging but in this talk, I'll look at a special class of symmetric algebras whose tilting theory is determined by a related hyperplane arrangement. This simple picture will then allow us to describe the stability manifold of such an algebra.

Thomas Brüstle (Université de Sherbrooke)

Abstract:

Lecture 1: g-vectors. The g-vectors originate from cluster theory, and there are by now many realizations of g-vectors in representation theory. We discuss first models by Derksen, Weyman, Zelevinsky using decorated representations, and move on to study support τ-tilting modules, two-term silting complexes, and projective presentations.
Lecture 2: Rudakov's definition of stability for abelian categories. Key point of this lecture is Rudakov's definition of stability for abelian categories. We plan to discuss the Kronecker quiver in some detail to explain where the Rudakov definition is more powerful: It allows to describe all torsion classes in the Kronecker example.
Lecture 3: From the g-vector fan to stability scattering diagrams. The plan is to show an embedding of the g-vector fan into the fan given by the stability scattering diagram.

Alastair King (University of Bath)

Classical Stability

Abstract:

Lecture 1: Weight stability. Semi-stability of quiver representations from GIT and the numerical criterion. Stability fan, including extension to real weights. Theta semistables form an abelian category.
Lecture 2: Slope stability. Relation to weight stability. Harder-Narasimhan filtrations and maximal destablising one-parameter subgroups. Central charge, slicing and theta-torsion theories. Stability scattering diagrams.
Lecture 3: Determinantal semi-invariants. Work of Schofield-Van den Bergh and Derksen-Weyman on semi-invariants for quivers and fin.dim. algebras, in terms of maps between projectives. Homological orthogonality in the quiver case.

Sven Meinhardt (University of Sheffield)

The art of counting semistable representations

Abstract: After introducing the concepts of moduli stacks and moduli spaces, we will face the problem of computing their cohomology. Luckily Donaldson-Thomas theory connects the cohomology of these two objects through a very nice formula if we restrict ourselves to semistable representations for a symmetric stability condition. We will also derive a wall-crossing formula which tells us that the restriction to semistable representations is not very restrictive et all. Along the way we will meet the Cohomological Hall (co)algebra which does not only categorify our counting problem but is also an interesting algebraic object for its own.

Lecture 1: Moduli stacks and spaces and their cohomology. I will illustrate why we need two concepts - moduli stacks and moduli spaces - in order to classify and to count representations. The Pros and Cons of introducing stability conditions will be discussed. We then move on to the cohomology of these two classifying objects. We start by introducing equivariant cohomology and finish with a few remarks about intersection cohomology. The first main result of Donaldson-Thomas theory for symmetric stability conditions (another Pro of stability conditions) will end the lecture.

Lecture 2: Vanishing cycles for quivers with potential. While the first lecture was mostly about quivers without relations, we will introduce some terminology to allow relations coming from a potential. More specifically, we provide a gentle introduction to vanishing cycle sheaves and discuss how the results of Lecture 1 have to be modified for quivers withpotential. If time permits, the technique of dimension reduction will be explained.

Lecture 3: Wall-crossing. As we have learned in the previous lectures, a good stability condition has to be chosen in order to make reasonable and interesting predictions. In the last lecture we will study the dependence on the chosen stability condition. As we shall see, a wall-crossing formula allows us to switch between various good stability conditions. If time permits, we will also discuss the (co)product of the Cohomological Hall algebra as it usessimilar ideas. The second main result of Donaldson-Thomas theory will end the lecture series.

Markus Reineke (Ruhr-Universität Bochum)

Stabilities and geometry of moduli spaces for quivers

Abstract:

Lecture 1: Deforming stabilities. Passing from a Theta-stability of a quiver to a deformed one induces a map of the corresponding moduli spaces of semistable representations whose geometry we will explore and apply to framed moduli spaces, resolutions of singularities, and probably Donaldson-Thomas invariants.

Lecture 2: Wall-crossing formulas. A change of Theta-stability of a quiver induces a prototypical wall-crossing formula relating the motives of the corresponding moduli stacks of semistable representations. We will derive this formula and discuss applications, for example the tropical vertex.

Lecture 3: Momentum polytopes. The Theta-stabilities admitting a semistable representation form the scalar part of the momentum polytope. This motivates the study of the full momentum polytopes of quivers. We will recall the symplectic reduction realization of moduli spaces of semistable quiver representations and derive a recursive description of momentum polytopes.

Invited talks

Hipólito Treffinger (University of Leicester)

Harder–Narasimhan filtrations from an algebraic perspective

Abstract: In this talk we show how chains induce Harder–Narasimhan filtrations on every non-zero object of an abelian category. Building on this result, we show several of their properties and we show their relation with stability conditions. Later, following ideas of Bridgeland, we show that all chains of torsion classes form a topological space with a natural distance and we will show how the stability manifold sits inside this space.
